

be separated, and I have endeavoured to show its migrations and gradual concentration as the work progresses, by tinting the fractions where it mostly would concentrate, the depth of tint representing the amount of concentration.

In the purest condition yet obtained victoria is an earth of a pale brown colour, easily soluble in acids. It is less basic than yttria and more basic than most of the earths of the terbia group. In chemical characters it differs in many respects from yttria. From a hot nitric acid solution victorium oxalate precipitates before yttrium oxalate and after terbium oxalate. On fractional precipitation with potassium sulphate the double sulphate of victorium and potassium is seen to be less soluble than the corresponding yttrium salt, and more soluble than the double sulphates of potassium and the terbium and cerium groups. Victorium nitrate is a little more easily decomposed by heat than yttrium nitrate, but the difference is not sufficient to make this reaction a good means of separating victorium and yttrium. Fusing the nitrates can, however, be employed advantageously to separate mixed victoria and yttria from the bulk of their associated earths.

On the assumption that the oxide has the composition Vc_2O_3 , the atomic weight of victorium is apparently not far from 117.

The photographed phosphorescence spectrum of victoria consists of a pair of strong lines at about λ 3120 and 3117; other fainter lines are at 3219, 3064, and 3060. Frequently the pair at 3120 and 3117 merge into one, but occasionally I have seen them quite distinct. The presence or absence of other earths has much influence on the sharpness of lines in phosphorescent spectra, and it is probable that these lines will be sharp and distinct when victoria is obtained quite free from its associates.

The best material for phosphorescing in the radiant matter tube is not the earth itself, but the anhydrous sulphate formed by heating the earth with strong sulphuric acid and driving off the excess of acid at a red heat. The sulphate thus produced, probably also containing some basic sulphate, is powdered and introduced into a bulb tube furnished with a quartz window, and a pair of thick aluminium poles sealed into the glass with stout platinum wires. The tube is well exhausted, keeping the current from a good induction coil going all the time. The pumping and sparking must continue until the earth glows with a pure light free from haze or cloudiness, and continues so to glow during the passage of the current without deterioration. The exposure in the spectrograph usually occupies an hour.

I give a diagrammatic plan of the two-prism spectrograph used in this research. It is furnished with two quartz prisms, quartz lenses and condensers. The slit jaws are of quartz, cut and polished according to the method I described in the *Chemical News*, vol. lxxi. p. 175, April 11, 1895.

The prisms are made in two halves according to Cornu's plan, one half of each being right-handed and the other half left-handed. One of the lenses also is right-handed and the other left-handed. By this device the effect of double refraction is so completely neutralised that with a five-prism instrument it is impossible, under high magnifying power, to detect any duplication of the lines.

The lenses are each of 52 mm. diameter and 350 mm. focus. The focus of the least refrangible rays is longer than that of the most refrangible rays, and the sensitive film must therefore be set at an angle to get the extreme rays into focus at the same time. But this alone is not sufficient. The focal plane is not a flat surface, but is curved, and the film must therefore be curved,¹ and it is only when both these conditions are fulfilled that perfectly

sharp images of spectral lines extending from the red to the high zinc line 2138.30 can be photographed on the same surface. Celluloid films are used, glass not being sufficiently flexible.

Using the middle position showing the whole spectrum on a plate, the angle is 40° , and the curvature is 190 mm. radius.

The condensers are of quartz, and are plano-cylindrical—one being double the focus of the other. The object of this, when spark-spectra are being photographed, is to concentrate on the slit a line instead of a point of light, as would be the case if ordinary lenses were used.

When photographing phosphorescent spectra—or, in fact, any spectra the wave-lengths of which are either unknown or require verification—I always photograph on the same film a standard spectrum, usually of an alloy of equal molecular weights of zinc, cadmium, tin and mercury. This forms a hard, somewhat malleable alloy, giving throughout the whole photographic region lines the wave-lengths of which are well known. The chief objection to this alloy is its volatility, the poles requiring frequent adjustment. Recently I have used pure iron for this purpose; this has the advantages of giving a great number of fine lines whose wave-lengths are accurately known, and not being very volatile, the poles do not rapidly wear away. If the poles are kept about 1 mm. apart, there is little or no interference from air lines.

The most simple method of applying the standard lines to an unknown spectrum is by the successive employment of two slightly overlapping diaphragms immediately behind the slit, one being used for the experimental and the other for the standard spectrum. In this way, without disturbing the instrument, the two spectra can be recorded on the plate one over the other; the overlap of 1 mm. being in the optical centre of the train. The resulting negative is then transferred to a micrometer measuring machine of special construction, having a screw of 1/100th of an inch pitch, and a means of accurately determining 1/1000th of its revolution, thus measuring directly to the 100/1000th of an inch. In this way, in a five-prism spectrograph having lenses 700 mm. focus, it is possible to determine wave-lengths of photographed lines to the sixth figure.

MATHEMATICS OF THE SPINNING-TOP.¹

I.

“TO those who study the progress of exact science, the common spinning-top is a symbol of the labours and the perplexities of men who had successfully threaded the mazes of the planetary motions. The mathematicians of the last century, searching through nature for problems worthy of their analysis, found in the toy of their youth ample occupation for their highest mathematical powers.

“No illustration of astronomical precession can be devised more perfect than that presented by a properly balanced top, but yet the motion of rotation has intricacies far exceeding those of the theory of precession.

“Accordingly we find Euler and D'Alembert devoting their talent and their patience to the establishment of the laws of the rotation of solid bodies. Lagrange has incorporated his own analysis of the problem with his general treatment of mechanics; and since his time Poinsot has brought the subject under the power of a more searching analysis than that of the calculus, in which ideas take the place of symbols, and intelligible propositions supersede equations” (Maxwell—“Collected Works,” I. p. 248).

Newton also cites the top as affording an experimental verification of his First Law of Motion—Lex. I. “. . .

¹ *Chemical News*, vol. lxxii. p. 87, August 23, 1895; and vol. lxxiv. p. 259, November 27, 1896.

¹ “Ueber die Theorie des Kreisel.” F. Klein und A. Sommerfeld. Heft i. ii. Pp. 196 and 197 to 512. (Leipzig: Teubner, 1897-8.)

Trochus, cujus partes cohaerendo perpetuo retrahunt sese a motibus rectilineis, non cessat rotari, nisi quatenus ab aere retardatur." We can translate *trochus* as a top, as well as a wheel or hoop.

But Newton lived too early to calculate any quantitative explanation of the theory of the top, and even his attempt at the simpler problem of precession was not a very great success. (*Principia*, lib. III., Prop. xxxix.) We had to wait for D'Alembert to straighten out the difficulties of first principles before arriving at an exact solution.

The key-note of the present treatise is found in § 2—"Analytische Darstellung der Drehungen um einen festen Punkt"—and in the bilinear relation in equation (5) on p. 25,

$$(1) \quad \lambda = \frac{\alpha\Lambda + \beta}{\gamma\Lambda + \delta}, \quad \text{with } \alpha\delta - \beta\gamma = 1,$$

expressing analytically the displacement of a rigid body about a fixed point.

If x, y, z are the coordinates of a point with respect to axes fixed in space, and X, Y, Z of the same point with respect to axes fixed in the body,

$$(2) \quad \lambda = \frac{x + yi}{r - z} = \frac{-r - z}{-x + yi},$$

while Λ is the same function of X, Y, Z ; thus λ and Λ play the part of a stereographic representation of the point on the sphere of radius r with respect to the poles in which the sphere is intersected by the axes Oz and OZ .

Expressed by Euler's unsymmetrical angles θ, ϕ, ψ ,

$$(3) \quad \alpha = \cos \frac{1}{2}\theta e^{\frac{1}{2}(\phi + \psi)i}, \quad \beta = i \sin \frac{1}{2}\theta e^{-\frac{1}{2}(\phi - \psi)i}$$

$$\gamma = i \sin \frac{1}{2}\theta e^{\frac{1}{2}(\phi - \psi)i}, \quad \delta = \cos \frac{1}{2}\theta e^{-\frac{1}{2}(\phi + \psi)i},$$

satisfying

$$\alpha\delta = \cos^2 \frac{1}{2}\theta, \quad \beta\gamma = -\sin^2 \frac{1}{2}\theta,$$

and

$$\alpha\delta - \beta\gamma = 1.$$

By putting

$$(4) \quad \alpha = D + Ci, \quad \beta = -B + Ai,$$

$$\gamma = B + Ai, \quad \delta = D - Ci,$$

so that

$$A^2 + B^2 + C^2 + D^2 = 1,$$

the versor-quaternion

$$(5) \quad Q = Ai + Bj + Ck + D$$

is obtained, which determines the displacement; this gives the authors an opportunity for an excursion into the Quaternion-Theory so far as required in their subject, and should delight the soul of Prof. Tait, always anxious to see more frequent applications of his favourite subject, to which allusion is made on p. 509.

In Maxwell's opinion, it is the introduction of the ideas, as distinguished from the operations and methods of Quaternions, which is valuable; and now we have McAulay's "Octonions" to assist, with plentiful illustrations of the dynamics of our subject.

The important use of the above bilinear relation between λ and Λ , for the treatment of the motion of a rotating body or a top, was pointed out by Prof. Klein in a lecture at Göttingen University in the winter-semester of 1895; the further development of the formulas was chosen by Prof. Klein as the subject of his Princeton Lectures in October 1896; and the present work is intended to be a complete presentation, with the collaboration of Prof. Sommerfeld.

Subsequent historical research, described on p. 511, has shown that the germs of similar ideas can be traced back through Hess (p. 429), Weierstrass (p. 511), up to Gauss in 1819. Gauss appears at the same time to have had some prophetic inspirations of the Quaternionic Theory, but as usual he carefully bottled up his ideas. It was said of the works of Friar Roger Bacon in the middle ages—"partim mutili direptis hinc inde quaterni-

onibus facti, tam raro comparent, ut facilius sit Sibyllae folia colligere quam nomina librorum quos scripsit"—and the same might be said of Gauss's unpublished manuscripts, now at length to be edited completely by the Göttingen Academy of Sciences, under the direction of Prof. Klein, the present occupant of Gauss's chair. We think then that Prof. Klein is too generous in renouncing the priority of his discovery, considering that he was the first to make the invention really work, and that his precursors allowed the germs of the idea to pass from them unfertilised, not perceiving their real importance.

In the special case of the symmetrical top the functions $\alpha, \beta, \gamma, \delta$ can be expressed by elliptic-theta functions; Prof. Klein calls them "multiplicative elliptic functions"; their form is given explicitly on p. 520, and now the solution is complete from the point of view of the mere mathematician of the school of—"Shut your eyes and write down equations."

"Mais il faut convenir que, dans toutes ces solutions on ne voit guère que des calculs, sans aucune image nette de la rotation du corps."

The authors, giving heed to this warning of Poincaré, devote the rest of the book to a careful examination and classification of the various cases which may occur; and also to what the ordinary mathematician in general so cordially detests, the working out and drawing in a diagram of some well-chosen numerical cases; only in this way is it possible to make sure of the accuracy and reality of the abstract formulas, and to lift mathematical science out of the arid collection of analytical results.

A first great requirement for the study of the movement of the top is an actual model, as shown on p. 1, or else a top such as that devised by Maxwell ("Collected Works," I. p. 248); sufficient rotation can be imparted by twirling the spindle by the finger; and a slight blow with the hand will give any desired variety to the pattern of the curve described by the end of the axle. A bicycle wheel, spinning in its ball bearings, supported in a fixed cup, would also serve the purpose.

As we have three independent constants at our disposal with the top, the number of possible cases is three-fold infinite (∞^3); and so the choice of a numerical case is at first sight an embarrassing one in its variety.

In the book the constants employed are such that (p. 223)

$$(6) \quad t = \int \frac{du}{\sqrt{U}}, \quad u = \cos \theta,$$

$$(7) \quad \psi = \int \frac{n - Nu}{A(1 - u^2)} \frac{du}{\sqrt{U}}$$

$$(8) \quad \phi = \int \frac{N - nu}{A(1 - u^2)} \frac{du}{\sqrt{U}} + N \left(\frac{1}{C} - \frac{1}{A} \right) t$$

$$(9) \quad A^2 U = (1 - u^2) (k - N^2 - 2APu) - (n - Nu)^2$$

$$= (1 - u^2) (k - n^2 - 2APu) - (N - nu)^2.$$

Here we must make the criticism that until p. 299 the mechanical interpretation of the quantities P, n, N is not very clearly defined so as to have their numerical values assigned in the C.G.S. system of units. Given the top we are to experiment with, we must first weigh it; denote the weight by W grammes; next measure the distance, h cm., between the C.G. and the point of support; then Wgh , dyne-cm., is the static moment denoted here by P , g denoting the acceleration of gravity (981) in cm./s.². The number A denotes, as usual, the moment of inertia, in g.cm.², of the top about an axis through its point perpendicular to its axis of figure; A can be determined experimentally by swinging the top as a plane or conical pendulum, and measuring the length of the equivalent pendulum, l cm., and the angular velocity m when swung without rotation as a conical pendulum of small aperture; then

$$l = \frac{A}{Wh}, \quad A = Whl = \frac{Pl}{g},$$

and

$$m^2 = \frac{g}{l} = \frac{Wgh}{A} = \frac{P}{A}.$$

So also C denotes, as usual, the moment of inertia, in g.cm.², about the axis of figure, and, with an angular velocity r rad./sec. about the axis of figure, $N = Cr$, the angular momentum about the axis; while n is the constant angular momentum round the vertical Oz .

In the discussion of the numerical cases, the authors start by taking a root $u = e$ of the cubic $U = 0$ as given, and then examining the various relations which subsist between N and n . But if one root of the cubic is known, the other two are determined by the solution of a quadratic, so that we may take all these roots as the three data of the question, and follow Darboux's method, as explained in the notes to Despeyroux' "Cours de Mécanique."

We may, for symmetry with Weierstrass' notation, denote the two extreme limits of θ by θ_2 and θ_3 , so that $\cos \theta_2$ and $\cos \theta_3$ are two roots of the cubic U ; the third root, being greater than unity, may be denoted by $\text{ch } \theta_1$, and now

$$A^2U = 2AP(\text{ch } \theta_1 - \cos \theta)(\cos \theta_2 - \cos \theta)(\cos \theta - \cos \theta_3) \\ = 2APV$$

suppose, with

$$\text{ch } \theta_1 > \cos \theta_2 > \cos \theta > \cos \theta_3.$$

It is not clear how any simplicity is gained by considering negative as well as positive values of P (p. 248); this seems to introduce needless complication in the classification, as we can take P always positive, and measure the angles θ_2 and θ_3 from the upright vertical position of the top. There may be a slight disadvantage in the case of the spherical pendulum, in which the chief part of the motion takes place in the lower hemisphere, but the counterbalancing advantages of simplicity of classification prevail on the whole.

In Darboux's representation of the motion of the axis by means of the generating lines of a deformable hyperboloid, we take a focal ellipse, of which the ratio of the axes is equal to the modulus κ of the real period; the co-modulus κ' of the imaginary period ω' is thus the eccentricity of the focal ellipse; and

$$(10) \quad \kappa^2 = \frac{\cos \theta_2 - \cos \theta_3}{\text{ch } \theta_1 - \cos \theta}, \quad \kappa'^2 = \frac{\text{ch } \theta_1 - \cos \theta_3}{\text{ch } \theta_1 - \cos \theta_2}.$$

The two generating lines HP and HP' are placed in position at an angle θ_3 as tangents to the focal ellipse, and the deformable hyperboloid is completed in Henrici's manner by a number of other rods as tangent lines, knotted together at the points of crossing. With this model we can represent graphically the various constants of the problem.

Returning to the case considered by the authors, where θ_3 , N , and n are given, we can select an arbitrary length OD , and measure off lengths HQ , HQ' along two straight lines inclined at an angle θ_3 , such that

$$(11) \quad \frac{HQ}{OD} = \frac{n}{2\sqrt{AP}}, \quad \frac{HQ'}{OD} = \frac{N}{2\sqrt{AP}};$$

and draw the perpendicular QO , $Q'O$ to the lines meeting in O . We are now given the conjugate semi-diameters OH and OD , by which the ellipse can be described through H , and the confocal ellipse, touching HQ and HQ' , is the focal ellipse of the deformable hyperboloid.

On this diagram

$$(12) \quad \cos \theta_3 = \frac{OH^2 - AB^2}{OD^2}, \quad \sin \theta_3 = 2 \frac{OS \cdot OM}{OD^2}, \\ \cos \theta_2 = \frac{OH^2 - OS^2}{OD^2}, \quad \sin \theta_2 = 2 \frac{OS \cdot ON}{OD^2},$$

$$\text{ch } \theta_1 = \frac{OH^2 + OS^2}{OD^2}, \quad \text{sh } \theta_1 = 2 \frac{OQ \cdot HP}{OD^2};$$

$$\frac{k}{2AP} = \frac{3OH^2 - AB^2}{OD^2} = \text{ch } \theta_1 + \cos \theta_2 + \cos \theta_3.$$

If θ_1 , θ_2 , θ_3 are given, then

$$e_{\theta_1} = \frac{HS'}{HS}, \quad \text{while} \quad \text{ch } \theta_1 = \frac{OH^2 + OS^2}{OH^2 - OS^2},$$

from which, when the focal ellipse is drawn, the position of H and the tangents HP , HP' can be drawn.

The angle AOQ is the amplitude function of a certain fraction fK' of Jacobi's quarter period K' , with respect to the co-modulus κ' , the eccentricity of the focal ellipse; denoting AOQ by ω , then, in Legendre's notation,

$$(13) \quad F\omega = fK',$$

whence the fraction f can be determined from his tables; so also the fraction f' for AOQ' .

In connection with the dynamical interpretation there is an important point L in the tangent HP , such that, in Jacobi's notation for the Zeta-function,

$$(14) \quad \begin{aligned} QL &= OAZfK' \\ LV, LT, LP &= OA(zs, zc, zd)fK'. \end{aligned}$$

Expressed in Legendre's notation

$$ZfK' = E\omega - fE,$$

from which the position of L can be calculated by Legendre's Tables; and now a reduction of the elliptic integrals of the third kind in (7) will show that the apsidal angle

$$(15) \quad \Psi = \frac{HL}{OA}K + \frac{1}{2}f\pi.$$

Integral (6) for the time t gives

$$(16) \quad mt = \int \frac{du}{\sqrt{2V}} = \frac{F\phi}{\sqrt{\frac{1}{2}(\text{ch } \theta_1 - \cos \theta_3)}} = \frac{OD}{OA}F\phi,$$

where

$$(17) \quad \cos \theta = \cos \theta_3 \cos^2 \phi + \cos \theta_2 \sin^2 \phi,$$

a different use of ϕ to that employed in (8).

Thus if T is the time taken by the axis to swing between the extreme inclinations θ_3 and θ_2 ,

$$(18) \quad mT = \frac{OD}{OA}K.$$

When H is at T , $N = n$, and the rosette curves described by Prof. Klein are obtained, in which the axis passes periodically through the highest vertical position, shown in Figs. 55, 56, 57. When H is at V , $N = n$ again, but now the axis describes an intermediate path, never becoming vertical. When H is at P , $N = -n$, and $\cos \theta_3 = -1$; the axis now describes a new series of rosette curves, all passing through the lowest vertical position. These rosette curves are shown in one of Mr. T. I. Dewar's stereoscopic diagrams.

If we make $\kappa = 1$, the focal ellipse becomes circular, and now $N = n = 2\sqrt{AP} \cos \frac{1}{2}\theta_3$, and the axis reaches the highest vertical position asymptotically, the case represented in Fig. 58; and the hyperboloid shuts up into the axis, like a deformable napkin-ring.

Having settled upon a certain modulus κ , and a certain fraction f , which gives the tangent TPQ of the focal ellipse, we can examine the variety of cases which arise by taking different positions of H on this tangent.

In the spherical pendulum $N = 0$, so that H must be placed on the pedal of the ellipse with respect to the centre O ; and now

$$(19) \quad \cos \theta_3 = -\frac{OQ}{OH} = -\frac{dn fK'}{dn f'K'};$$

and similarly

$$\cos \theta_2 = -\frac{cn fK'}{cn f'K'}, \quad \text{ch } \theta_1 = \frac{sn fK'}{sn f'K'}.$$

When H is on the tangent at B or B', the curve described by points on the axis have cusps; when H lies between these points, the curves are looped, and the associated herpolhode has points of inflexion.

In steady precessional motion $\kappa=0$, and the focal ellipse coalesces with the line SS'; now $f=0$ and $K=\frac{1}{2}\pi$, so that the apsidal angle

$$(20) \quad \Psi = \frac{HS}{OS} \frac{1}{2}\pi.$$

If μ denotes the constant angular velocity of precession,

$$\mu T = \Psi, \quad mT = \frac{OD}{OS} \frac{1}{2}\pi,$$

so that

$$(21) \quad \frac{\mu}{m} = \frac{HS}{OD}.$$

The steady motion relations

$$(22) \quad P = N\mu - A\mu^2 \cos \alpha, \quad n = A\mu \sin^2 \alpha + N \cos \alpha,$$

when α denotes, instead of θ_3 , the constant inclination of the axis to the vertical, will be found, on eliminating μ , to be equivalent to the geometrical relation

$$(23) \quad OQ \cdot OQ' = \frac{1}{4}OD^2 \sin^2 \alpha,$$

so that O lies on a certain hyperbola, $xy = \frac{1}{4}OD^2$, referred to HQ, HQ' the asymptotes as axes; thus given N and α , a geometrical construction will determine the solution of the problem.

Having selected OD arbitrarily, the hyperbola is drawn, and then having laid off HQ' to scale, draw Q'O at right angles to the asymptote HQ' to meet the hyperbola in O; the tangent to the hyperbola at O will meet the asymptotes in S and S'.

The second solution will depend on the second point O' in which Q'O cuts the hyperbola. To realise the state of motion shown in Fig. 28, the point O on the hyperbola must be close to the asymptote.

It is very desirable that a penultimate case of this nature should be worked out completely. We have the requisite analysis for the calculation of the algebraical case when there are 22 cusps, but the work would require the arithmetical courage of a Dewar, now unfortunately no longer available.

Supposing that in consequence of the friction of the pivot the motion has steadied down to uniform precession, then a (following) tap on the spindle, to (hurry) the precession makes O move along OQ and produces a focal (ellipse) and the axis (rises).

A tap in the vertical plane makes O start out normally to the plane, and the lines HQ, HQ' are now generating lines of Darboux's hyperboloid, in an intermediate position.

Once the limits θ_2 and θ_3 are assigned, and the corresponding apsidal angle Ψ , a regular curve satisfying these conditions drawn empirically will give an idea of the complete curve described by the axis.

But if it is desired to plot a number of intermediate points from tabular matter of the elliptic functions, we are baffled by the mixture of the real and imaginary arguments of the theta-functions required in the calculation of ψ and ϕ , although θ is readily calculated by equation (17). A courageous attempt at this computation is made in IV. 8, by Herr Blumenthal, but the hidden rocks of error are numerous and plentiful enough to make this procedure dangerous. If the calculation of a number of guiding points between the extreme points of a branch of the spherical curve is required, we had better utilise Jacobi's theorem, that the curve described in a horizontal plane round the vertical OG by the extremity of the vector OH of resultant angular momentum is a Poinot herpolhode; and having plotted this herpolhode, the vertical plane

defined by ψ may be drawn perpendicular to the tangent HK of the herpolhode, while a simple relation of the form

$$(24) \quad \cos \theta + 2 \frac{OH^2}{OD^2} = \frac{k}{2AP} = \text{ch } \theta_1 + \cos \theta_2 + \cos \theta_3,$$

will give the corresponding value of θ , and hence the spherical curve, or its projection, orthographic or stereographic, can be plotted to any desired accuracy.

This theorem of Jacobi is almost self-evident when interpreted by means of Darboux's representation of the motion by the deformable articulated hyperboloid.

In this method the rod OG is held in the vertical position, and the point H is guided round the herpolhode; and then OG, connected by the articulation, will imitate the movement of the axis of the top; for a quadric surface can be drawn through H coaxial with the articulated hyperboloid, and normal to HP at H, and the squares of the semi-axes of this quadric will be, by a well-known theorem of solid geometry

$$(25) \quad HQ \cdot HV, HQ \cdot HT, HQ \cdot HP,$$

with proper attention to sign; and these being fixed lengths, the quadric is a fixed quadric.

Its equation may be written

$$(26) \quad Ax^2 + By^2 + Cz^2 = D\delta^2,$$

with

$$(27) \quad \delta = HQ; \quad \frac{D}{A} = \frac{HV}{HQ}, \quad \frac{D}{B} = \frac{HT}{HQ}, \quad \frac{D}{C} = \frac{HP}{HQ} (\pm),$$

the $\pm$ sign being taken according as Q and V, T, P are on the (same) (opposite) sides of H.

An accent will serve to distinguish the case where a fixed quadric rolls on the plane through H perpendicular to HP; and now we see why the same polhode described by H with respect to the axis of the articulated hyperboloid has associated with it two distinct herpolhodes.

(To be continued.)

LIFE-HISTORY OF THE PARASITES OF MALARIA.

THE parasites which cause malarial fevers in human beings belong to a very homogeneous group, other species of which are found in certain bats and birds. The life-history of the three species of this group which have been completely studied is as follows. The youngest parasites exist as amœbulae or myxopods within the red blood corpuscles of the vertebrate host. Each amœbula possesses a nucleus and nucleolus; and its movements vary in extent and rapidity with the species, but, in the case of birds, never encroach upon the nucleus of the corpuscle. The amœbulae increase in size; and, as they do so, tend to lose their movements and to accumulate in their ectoplasm certain black granules, the pigment or melanin, which are the product of assimilation of the hæmoglobin of the corpuscle. In from one to several days the parasites reach their highest development within the vertebrate host, and become either (a) sporocytes or (b) gametocytes.

The sporocytes, which are produced asexually, contain spores which vary in number according to the species. The spores do not possess any appreciable cell-wall. When they are mature the corpuscle which contains them bursts and allows them to fall into the serum. They then attach themselves to fresh blood corpuscles, and continue the propagation of the parasites indefinitely in the vertebrate host. The residuum of the sporocyte, consisting chiefly of the pigment, is taken up by the phagocytes of the host for eventual disposal in the host.

The gametocytes, while in the blood of the vertebrate host, are still contained in the shell of the corpuscle.